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SEMILEPTONIC DECAYS AND CKM MATRIX ELEMENTS IN A TCF

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Abstract

We argue that a Tau-Charm factory (TCF) could improve our knowledge of CKM matrix elements since it is an incomparable tool to check the models and methods applied to extract V_{bu} from B decay partial widths. We report on some recent proposals to improve on parton model. Turning to exclusive decays, we compare the predictions from quark models, QCD sum rules, effective Lagrangians and lattice QCD. Quark models have anticipated on heavy quark symmetry. Their difficulty to account for the q^2 dependence might be partly cured by relativistic corrections. QCD sum rules and lattice seem to disagree on the q^2 dependence of axial form factors. We discuss the extrapolation from D to B . Present uncertainties do not allow to predict reliably the $B \rightarrow \pi, \rho l \nu$ matrix elements. We argue that QCD sum rules are in a good position to study q^2 dependence down to $q^2 = 0$. Lattice QCD is able to check the heavy quark scaling laws for the heavy to light semileptonic decays. It seems to confirm the increase of A_2/A_1 from D to B meson predicted by the scaling laws. Finally semileptonic decays in a TCF would give incomparable informations on $K\pi$ phase shifts, and on K^{**} resonances.

1 INTRODUCTION

During our working group on CKM matrix elements and semileptonic decays, we heard five interesting talks. Four of them were dealing mainly with heavy flavours semileptonic decays: by P. Roudeau on experimental results, by P. Colangelo on QCD sum rules, by V. Lubicz on lattice QCD and by N. Di Bartolomeo on an effective Lagrangian approach. The fifth talk, by M. Shifman, was more generally advocating a new operator expansion method for heavy flavours. We will

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try to summarise these enlightening contributions, and add some comments on quark model studies of heavy flavours semileptonic decays. Semileptonic decays of heavy mesons have attracted considerable interest in the past years as they play a crucial role in the determination of the Cabibbo-Kobayashi-Maskawa mixing matrix. Now the question is: what could a Tau-Charm factory (TCF) learn us about CKM angles ? At first sight the answer seems rather negative. A TCF would mainly give a direct access to V_{cs} and V_{cd} . It is true that these angles are not directly known to a high degree of accuracy, but unitarity constrains them very strongly from V_{us} and V_{ud} . Nobody would bet a cent on the chances of something new to happen here. It is well known that the CKM angles we are mostly interested in, are those of the third generation. And to know them we need a b -factory. What is then the use of a TCF in that respect ? The fact is that a b -factory gives you an experimental number, say a partial width, $\Gamma = |V_{ub}|^2 X$, where X is, up to kinematical factors, the squared matrix element of some operator between hadrons. X is not given by any symmetry principle (except in one point of phase space), X is in general a very difficult quantity to compute theoretically. Any model, any theoretical method that makes predictions about the B decay matrix elements has something to say also about charm decay and can be checked there since we know the relevant CKM angles. This point is crucial, *a good understanding of charm physics is an unescapable necessity to make reliable predictions concerning beauty*. The belief that heavy Quark Effective Theory (HQET) could save us this effort is far too naive. Heavy Quark Symmetry is a very useful tool, but it does not provide us with the needed quantities (we will see some examples in the following). It is also fair to say that the extrapolation from charm to beauty is by no means trivial and needs a lot of theoretical work. One major theoretical challenge to physics today is to improve our understanding of the non-perturbative aspects of QCD. M. Shifman expressed the view that a TCF would be a new and powerful microscope into strong interactions. Indeed the round table of this workshop has stressed that a TCF could increase dramatically the accuracy of experimental data related to non-perturbative QCD. Mutatis mutandis, this could be compared to the role of LEP as related to the perturbative aspects of the standard model. Many illustrations of what a TCF could bring in this respect have been produced during this workshop. The precise measurement of hadronic matrix element of $\Delta C = 1$ currents, that will be achieved from semileptonic decays, is another example of what a TCF can learn us about non-perturbative QCD. Finally, although this is not too popular at present, we should not forget that the analysis of non charmed final states of semileptonic decays would be a rich source of informations.

1.1 Vector meson dominance and scaling laws.

From Lorentz invariance, it is possible to express in all generality the current matrix elements as:

$$\langle K | J_\mu | D \rangle = \left(p_D + p_K - \frac{M_D^2 - M_K^2}{q^2} q \right)_\mu f_K^+(q^2) + \frac{M_D^2 - M_K^2}{q^2} q_\mu f_K^0(q^2) \quad (1)$$

$$\begin{aligned} \langle K_r^* | J_\mu | D \rangle &= e_r^\beta \left[\frac{2V(q^2)}{M_D + M_{K^*}} \epsilon_{\mu\gamma\delta\beta} p_D^\gamma p_{K^*}^\delta + i(M_D + M_{K^*}) A_1(q^2) g_{\mu\beta} \right. \\ &\quad \left. - i \frac{A_2(q^2)}{M_D + M_{K^*}} P_\mu q_\beta + i \frac{A(q^2)}{q^2} 2M_{K^*} q_\mu P_\beta \right], \end{aligned} \quad (2)$$

where q is the momentum transfer, $q = p_D - p_K$ or $q = p_D - p_{K^*}$, $P = p_D + p_{K^*}$ and e_r^β is the polarization vector of the K^* . $f_K^{+,0}$, V , $A_{1,2}$ and A are dimensionless form factors. For the axial current, some authors prefer another set of form factors:

$$\begin{aligned} f(q^2) &= (M_D + M_K) A_1(q^2), \\ a_+(q^2) &= -i \frac{A_2(q^2)}{M_D + M_K}, \quad a_-(q^2) = i \frac{A(q^2)}{q^2}. \end{aligned} \quad (3)$$

Let us consider the kinematics of $P \rightarrow M l \nu$, where P is D or B and M is a final light meson, K , K^* , π or ρ . The physical region for semileptonic decay corresponds to $0 \leq q^2 \leq q_{max}^2$ where $q_{max}^2 = (M_P - M_M)^2$. q_{max}^2 corresponds to the no recoil point, i.e. the final meson as well as the dilepton system are at rest in the initial restframe²: $\vec{q} = 0$. This point is the closest to the nearest t -channel pole, located at $q^2 = M_t^2$ where M_t is the mass of the lightest meson with the quantum numbers exchanged in the t -channel. For example when the final meson is K or K^* , the exchanged meson is a $c\bar{s}$ meson, vector, axial or scalar meson according to the cases. It is known that $M_t - M_P \rightarrow \text{constant}$ when $m_Q \rightarrow \infty$ ³. It follows $M_t^2 > M_P^2 > q_{max}^2$, but $M_t - \sqrt{q_{max}^2}$ is not too large and stays constant when the $M_P \rightarrow \infty$. As a consequence, *the region near q_{max}^2 may feel strongly the influence of the nearest pole*⁴. On the contrary, when q^2 decreases, the distance to the nearest pole increases, and the relative influence of all the other singularities in the t -channel increases, *washing out the dominance of the nearest pole*. Isgur and Wise[1] have proposed a set of scaling laws relating semileptonic amplitudes for a given final light meson (K, K^* or π, ρ) when the mass of the initial meson goes to infinity: up to $O(1/M_P^2)$, up to logarithmic corrections, one expects the

²All over this report we will chose as reference frame the rest frame of the initial heavy meson.

³ m_Q is the heavy quark mass.

⁴This is usually phrased, in the case of the form factor f^+ , as the vector meson dominance (VMD) hypothesis.

following behaviour for the relevant form factors[1]:

$$\begin{aligned} \frac{f^+}{M_P^{1/2}} &= \gamma_+ \times \left(1 + \frac{\delta_+}{M_P}\right) & \frac{V}{M_P^{1/2}} &= \gamma_V \times \left(1 + \frac{\delta_V}{M_P}\right) \\ \frac{A_2}{M_P^{1/2}} &= \gamma_2 \times \left(1 + \frac{\delta_2}{M_P}\right) & A_1 M_P^{1/2} &= \gamma_1 \times \left(1 + \frac{\delta_1}{M_P}\right) \end{aligned} \quad (4)$$

where M_P is the mass of the initial heavy meson. The expansions given in eqs.(4) become valid in the limit of large m_Q , at fixed momentum $-\vec{q}$ of the light meson (in the frame where the heavy meson is at rest) and when $|\vec{q}| \ll m_Q \sim M_P$. The above conditions are always satisfied for q_{max} , when the initial and final mesons are both at rest. For $q^2 = 0$, $|\vec{q}| = (M_P^2 - M_M^2)/(2M_P) \sim M_P/2$. Assuming, for instance, that we acquire a good knowledge of $D \rightarrow \rho$, the scaling laws in eq. (4) will lead to a prediction of $B \rightarrow \rho$ in the region $|\vec{q}| < M_D/2$. This is only a small region close to q_{max}^2 in the physical phase space for $B \rightarrow \rho$, very far from $q^2 = 0$, the latter being the dominant contribution to phase space. The conclusion is that *the scaling laws are not enough* to allow an extrapolation from $D \rightarrow \rho$ to $B \rightarrow \rho$. A further extrapolation in q^2 from the small recoil region down to $q^2 = 0$ is necessary. And this cannot be done reliably from the simplest use of nearest pole dominance, as we have argued in the preceeding paragraph. *This extrapolation is a formidable challenge to theory in the coming years.*

2 INCLUSIVE SEMILEPTONIC DECAYS

At present the only positive experimental source of information concerning V_{ub} comes from inclusive semileptonic B meson decay, since ARGUS and CLEO do not agree on the existence of a positive signal in exclusive decays. These inclusive decays are compared, either to the parton model, or to the sum of exclusive decays in some exclusive model. We will discuss the exclusive models later on. The parton model, relying on duality, directly predicts the inclusive decays from an amplitude computed with free final quarks, including perturbative QCD corrections. As a first approximation the parton model is very good. However an improvement of its accuracy is not easy since its predictions depend very strongly on the b quark mass, on the B meson wave function and because the corrections to duality are not known. One needs a systematic approach able to improve from the parton model. Such a systematic non perturbative approach has been proposed by Bigi, Shifman, Uraltsev and Vainshtein[2]. Their idea is a generalisation of operator expansion, it amounts to expand the heavy quark weak interaction *in the background of the light quark and gluon fields* into a series of local operators. It is an expansion in $1/m_Q$ (inverse of the heavy quark mass), the non trivial contributions starting at $(1/m_Q)^2$. This is a very appealing method, but up to now only semi-quantitative. Tests are needed, and the charm sector seems the

best place since the $1/m_c^2$ corrections are large (maybe even too large in some cases). However an unlucky feature is that, due to singular contributions, the operator expansion fails near the end point of the electron spectrum. This happens to be the point where the suppression of the charm background allows an experimental observation of $b \rightarrow u$ semileptonic decay. These singularities also prevent to predict the energy spectrum in the case of charm decay, only integrated quantities being predictable. We are thus lead to turn toward exclusive theoretical calculational methods in order to estimate what degree of accuracy may be expected from them.

3 QUARK MODELS

It must first be emphasized that in contrast to lattice QCD (and partly to QCD sum rules), quark models (QM) most often do not provide constraining predictions. They are not definite approximations of QCD and their predictions depend on various parameters (quark masses, potentials) which have no direct physical meaning and make sense only within these specific models. These parameters can be fixed independently through spectroscopy, but only roughly. Exceptions to the lack of constraining predictions are for example the heavy quark symmetry relations, but they have been shown to derive from much more general principles. Being interested in semileptonic decays, the main interest of quark models lies in the *physical insight* they provide through the dynamical concepts of composite systems. However the position and reliability of quark models is strongly dependent on the channel and decreases when going from $B \rightarrow D, D^*$ transitions, to $D \rightarrow K, K^*$ or $D \rightarrow \pi, \rho$ decays, not to speak of $B \rightarrow \pi, \rho$ which presents very large momentum transfers.

3.1 Quark models and Isgur-Wise relations.

Quark models, even in a rather naive stage, give straightforwardly the probable pattern of magnitude of the various form factors, which for the ground state transitions $0^- \rightarrow 0^-, 1^-$ already amount to six a priori independent ones. This simple pattern has been first put forward by Altomari and Wolfenstein[3] who have discovered the simplificatory virtues of α) having a light spectator with heavy flavor transition, β) working near q_{max}^2 (the no recoil point). Most predictions derive from the non-relativistic quark model (NRQM). An exception is A_2 (a_+), crucial for the polarization in $0^- \rightarrow 1^-$ decays, which can be derived consistently from a subtle treatment including Wigner rotations[4]. The general predicted pattern at q_{max}^2 is in *qualitative* agreement with what is known experimentally⁵. The agreement seems now to include also A_2 (a_+) which has once been controversial. *Quantitatively* everything seems OK for $B \rightarrow D, D^*$ (taking

⁵Unhappily, due to phase space, experimental measurements are rather taken near $q^2 = 0$

into account the uncertainty on V_{cb}). Turning now to heavy to light semileptonic decays ($B, D \rightarrow K, K^*, \pi, \rho$), NRQM predictions are not very different from those of the heavy to heavy case. As a particular consequence, the scaling relations of eq. (4) follow straightforwardly. But NRQM goes further, it predicts also relations between vector and axial form factors, similar to those predicted for the heavy to heavy case. These relations are not confirmed by experiment: $\Gamma(D \rightarrow K^*)/\Gamma(D \rightarrow K)$ is predicted twice too large.

The huge progress made by Isgur and Wise and by a sery of others authors[5], [6] was to demonstrate three things: 1) the above quark model predictions for heavy to heavy transitions at q_{max}^2 are in fact exact in QCD in the infinite quark mass limit, 2) in the same limit exact relations between all the heavy to heavy form factors at $q^2 \neq q_{max}^2$ for $0^-, 1^- \rightarrow 0^-, 1^-$ can also be derived from QCD 3) the scaling law in eq. (4) can also be derived on general grounds from QCD, provided $|\vec{q}| \ll M_P$. Most current models would not satisfy the second set of relations (see for example ref. [6]). It has been shown that a careful *relativistic* treatment of spins for states in motion allows to construct quark models satisfying automatically these relations[7].

3.2 Relativity in the calculation of absolute magnitudes of form factors

With the advent of heavy quark symmetry (HQS) machinery, the appeal of quark models is partly lost, since the above simple relations can be derived in an exact QCD framework. The quark model has still the interest of offering a very intuitive realisation of HQS. However the main interest is displaced towards those features which HQS do not predict. These are: a) the absolute magnitudes of the form factors away from the no recoil point, b) the corrections to HQS, subleading in $1/m_Q$ (m_Q is the heavy quark mass), which in HQS depend on a host of new arbitrary functions. We will not consider point b) in detail since a) is the main challenge. It will be sufficient to say a word about corrections to HQS relations at q_{max}^2 . They are known[8] to be of order in $1/m_Q^2$. They seem to be dominated by the Dirac spinors relativistic corrections to the axial current. There are also corections to all the currents fom the overlap of wave functions which differ from 1 if the heavy quark masses are finite. But they are small, entering through the expression $(1/m_Q - 1/m'_Q)^2$ due to a sort of Ademollo-Gatto theorem. The relativistic correction reduces the axial current contribution, in the right direction to lessen the above mentioned discrepancy of NRQM for $\Gamma(D \rightarrow K^*)/\Gamma(D \rightarrow K)$, while its being very small for $\Gamma(B \rightarrow D^*)/\Gamma(B \rightarrow D)$ explains the good NRQM prediction⁶. Let us concentrate on point a). We start

⁶In the case of $D \rightarrow K, K^*$ the relativistic “corrections” are more than corrections, since the final meson are light. We use here these corrections to get a qualitative hint, assuming that the first order gives the right trend

Ref.	$f_K^+(0)$	$V(0)$	$A_1(0)$	$A_2(0)$
Lat. [26]	0.65 ± 0.18	0.95 ± 0.34	0.63 ± 0.14	0.45 ± 0.33
Lat. [21]-[23]	0.63 ± 0.08	0.86 ± 0.10	0.53 ± 0.03	0.19 ± 0.21
Lat. [24]-[25]	0.90 ± 0.08 ± 0.21	1.43 ± 0.45 ± 0.49	0.83 ± 0.14 ± 0.28	0.59 ± 0.14 ± 0.24
QM [12]	0.76	1.23	0.88	1.15
QM [11]	$0.76 - 0.82$	1.1	0.8	0.8
SR [19]	0.6 ± 0.10	—	—	—
SR [17]	$0.6^{+0.15}_{-0.10}$	1.1 ± 0.25	0.5 ± 0.15	0.6 ± 0.15
Exp. [28]	0.70 ± 0.08	0.9 ± 0.3 ± 0.1	0.46 ± 0.05 ± 0.05	0.0 ± 0.2 ± 0.1

Ref.	$A_1(q_{max}^2)$	$V(0)/A_1(0)$	$A_2(0)/A_1(0)$	$f^0(q_{max}^2)$
Lat. [26]	0.62 ± 0.09	1.50 ± 0.28	0.7 ± 0.4	0.93 ± 0.13
Lat. [23]	0.77 ± 0.20	1.6 ± 0.2	0.4 ± 0.4	—
Lat. [25]	$1.27 \pm 0.16 \pm 0.31$	$1.99 \pm 0.22 \pm 0.33$	$0.7 \pm 0.16 \pm 0.17$	—
QM [3]	—	1.9	0.8	—
QM [14]	—	1.0	1.0	—
QM [12]	—	1.1	1.3	1.15
QM [11]	—	1.4	1.0	—
SR [17]	—	2.2 ± 0.2	1.2 ± 0.2	—
Exp. [28]	0.54 ± 0.06 ± 0.06	—	—	—
Exp. [29]	—	2.00 ± 0.33 ± 0.16	0.82 ± 0.23 ± 0.11	—

Table 1: *Semileptonic form factors for $D \rightarrow K$ and K^* . “Lat.” refers to lattice QCD, “QM” to quark models, “SR” to QCD sum rules and “Exp.” to experiment.*

from the observation that the naive NRQM fails *qualitatively* in two ways: i) the slope of form factors seems much too small at *small* $|\vec{q}|$ ($q^2 \sim q_{max}^2$) ii) the form factors have on the contrary a much too steep falloff at *large* $|\vec{q}|$. These facts have been well known since very long and have already been discussed in ref. [9]. The smallness of the predicted slope at q_{max}^2 can be appreciated in the heavy quark limit by the prediction[10] of the factor ρ^2 (minus the slope of the Isgur Wise function at the origin). It is unambiguously predicted by the NRQM to be $\rho^2 = m_d^2 R^2/2$, where m_d is the spectator quark constituent mass and R is a radius⁷, rather safely related to the spectrum, which should not be very different from $R^2 = 6 \text{ GeV}^{-2}$ in the heavy quark limit. This gives something like $\rho^2 = 0.3$, in contrast to the roughly measured $\rho^2 > 1.0$ from $B \rightarrow D^* l \nu$. The too steep falloff at large $|\vec{q}|$, particularly dramatic in $D \rightarrow \pi$ and even more in $B \rightarrow \pi, \rho$ where large momentum transfers are kinematically allowed, depends on the wave function and is particularly striking for gaussian wave functions. Of course this failure of the naive quark model is very serious: it is qualitative and it concerns what seems to be the specific domain left to quark models. How do they try to escape this disappointing situation ? Apart from the sery of purely adhoc recipes used in ref. [11]⁸ we observe two main theoretical trends:

- 1) *Nearest pole dominance.* Many authors simply renounce to predict the form factors from the quark model except at one point ($q^2 = 0$ or $q^2 = q_{max}^2$) and prefer to use nearest pole dominance. This is done for example in refs. [3] and [12]. We have already discussed in the subsection 1.1 the validity of the nearest pole dominance approximation. Let us simply observe that it is not obvious to explain why one can combine these two different approaches and assume that the quark model is still valid at some q^2 , while the nearest t -channel pole is dominating at that same point.
- 2) *Relativistic center-of-mass motion effects.* This is a very old idea, yet there is not, up to now, a systematic relativistic treatment of semileptonic decays. Rather several interesting ideas have been put forward, which have still to be connected together.

At large q^2 , the Lorentz contraction effect has since long been known to smoothen the gaussian falloff[13]. This effect is included in the quark model of ref. [12] and explains why it does not predict too small values at $q^2 = 0$ even for $B \rightarrow \pi$ ($f_\pi^+(0) \sim 0.3$). However, in our opinion[7], the relativistic boost of spins counterbalances this by large depression factors and still leads to very small values⁹. At small q^2 , the latter relativistic effects of spin [7] seem happily to enlarge ρ^2 with respect to its “static” value $m_d^2 < \vec{r}^2 > /3$, where m_d is the spectator quark

⁷The ground state wave function radius for the harmonic oscillator potential.

⁸Notwithstanding these recipes the model of ref.[11] leads to an extremely small result for $B \rightarrow \pi, \rho$, see table 4, because the falloff still remains gaussian.

⁹The authors of ref. [12] omitted to consider this effect.

constituant mass and the average is taken over the rest frames wave functions. Recently an important progress has been made by Close and Wambach[10] who found a larger additional contribution to ρ^2 due to the Lorentz transformation of the *spatial* wave function. This effect seems specific to the situation where $m_d \ll m_Q$. It would not be present with the simple Lorentz contraction prescription. Quantitative predictions are however hampered, according to us, by the fact that $\langle \vec{r}^2 \rangle$ cannot be identified with the non relativistic radius: it is submitted to relativistic binding corrections of the same order as the ones under discussion. In conclusion, the introduction of relativity in the quark models seems promising in many respects, but it would stand on a more solid ground for $b \rightarrow c$ decays than for $c \rightarrow s$, where nearest pole dominance is not to be excluded (see sections 4 and 6). Moreover, $D \rightarrow \pi$ and $B \rightarrow \pi$ seem to escape the possibilities of quark models. The numerical predictions from quark models are reported, in comparison with others results, in tables 1, 3 and 4.

4 QCD SUM RULES

We will not give too many details on QCD sum rules methods since they have been discussed by Guido Martinelli [15] in his talk on leptonic decay constants. For further details we refer to ref. [16]. The theoretical basis of QCD sum rules is totally rigourous: it incorporates analiticity, asymptotic freedom, and non pertrubative effects are implemented through vacuum expectation values of some operators: $\langle \bar{q}q \rangle$, $\langle G_{\mu\nu}G^{\mu\nu} \rangle$ and $\langle \bar{q}\sigma_{\mu\nu}G^{\mu\nu}q \rangle$. However, the practical use of QCD sum rules encounters important difficulties: 1) The matching between the perturbative and non perturbative domains is by no means trivial, and in practice it depends on a parameter usually labelled s_0 the value of which is to some extent arbitrary; 2) the estimate of the vacuum expectation values of the condensates are estimated from other applications of QCD sum rules, but with large uncertainties; 3) there is a strong dependence on the heavy quark mass. Compared to quark models, QCD sum rules do not have any difficulty with Lorentz covariance: the treatment is covariant from the start. Even more important, this technique is by no means restricted to one value of q^2 . It can be applied to any q^2 except the vicinity of q_{max}^2 and is therefore the *tool to study the q^2 dependence of the form factors*. To our knowledge, this study has only been performed in refs. [17] and [18]. Their conclusion is, for D as well as B meson decays, that the vector form factors dependence on q^2 is compatible with the nearest vector meson pole dominance, but that *the axial currents do not show anywhere any effect of the nearest axial meson pole*. The latter conclusion is quite a surprise, in contradiction with lattice QCD as we shall see. More work is needed here to understand this point thoroughly. The numerical predictions from QCD sum rules are reported in tables 1, 3 and 4.

Figure 1: *We show an example of the q^2 behaviour of the form factors. We have chosen quark masses close to those involved in the $D \rightarrow K, K^*$ decays.*

5 EFFECTIVE LAGRANGIAN

The authors of ref. [20] use an effective Lagrangian that incorporates both heavy quark symmetry and chiral symmetry. Besides light ($q\bar{q}$) and heavy-light ($Q\bar{q}$) pseudoscalar mesons, they incorporate light vector mesons and heavy-light scalar mesons in their Lagrangian. The parameters of the model are tuned to $D \rightarrow \pi$ and $D \rightarrow K^*$, from which they predict $D \rightarrow \pi, \rho$ and $B \rightarrow \pi, \rho, \dots$. This is equivalent to extrapolating from D to B through the scaling laws of ref. [1], neglecting the $O(1/m_Q)$ corrections, i.e. applying eq (4) with $\delta_+ = \delta_V = \delta_1 = \delta_2 = 0$. The latter corrections may nevertheless not be negligible as indicated by lattice calculations. The resulting form factors are reported in table 4.

6 LATTICE QCD.

Here again we will not repeat the general description of lattice QCD method by Guido Martinelli. Lattice QCD is based on first principles but suffers from several practical limitations. Among these limitations, the fact that the cut-off (the inverse lattice spacing) is rather low: a^{-1} ranges from 2 to 4 GeV. The consequence is that the masses of the “heavy” quark has to verify $m_Q \ll a^{-1}$.

\vec{p}	$\gamma_+ \text{ GeV}^{-1/2}$	$\gamma_V \text{ GeV}^{-1/2}$	$\gamma_1 \text{ GeV}^{+1/2}$	$\gamma_2 \text{ GeV}^{-1/2}$
(0, 0, 0)	—	—	0.96 ± 0.16	—
(1, 0, 0)	0.39 ± 0.25	0.29 ± 0.12	1.05 ± 0.25	0.44 ± 0.25
\vec{p}	$\delta_+ \text{ GeV}$	$\delta_V \text{ GeV}$	$\delta_1 \text{ GeV}$	$\delta_2 \text{ GeV}$
(0, 0, 0)	—	—	-0.33 ± 0.09	—
(1, 0, 0)	0.0 ± 1.1	1.9 ± 1.3	-0.46 ± 0.22	-0.6 ± 0.8

Table 2: *The coefficients of the $1/m_Q$ expansion of the form factors defined in eqs.(4).*

This allows to study the charm quark, but for beauty an extarpolation is needed, sometimes combined with direct data for infinitely massive quarks. Speaking of semileptonic decays, assuming that we work in the rest frame of the initial meson, one has to vary q^2 , i.e. $-\vec{q}$, the momentum of the final meson. But the finite volume of the lattice allows only a discrete set of values for the momentum: $\vec{q} = (n_x, n_y, n_z)2\pi/L$ where L is the spatial length of the lattice and n_x, n_y, n_z are integers. In practical calculations, L ranges from 1 to 2 fermis. A further constraint is that $|\vec{q}| \ll a^{-1}$. This means that only a few points are computable, and the errors increase fast with $|\vec{q}|$. Several calculations have been performed: refs. [21]-[26]. Taking as an example the most recent one, ref. [26], we notice that the discreteness of momenta is not a big problem in the case of $D \rightarrow K, K^*$, because the parameters are such that the besides the zero momentum point ($q^2 = q_{max}^2$), the point at $\vec{q} = (1, 0, 0)2\pi/L$ happens to be located close to $q^2 = 0$. Thus lattice calculations have direct access to both ends of the physical region. In figure 1 taken from ref. [26] we compare the lattice points to the nearest pole dominance, with the position of the pole computed from the lattice. As can be seen, within large errors, lattice QCD is compatible with nearest pole dominance in the $D \rightarrow K, K^*$ decay. This seems to disagree, as far as axial currents are concerned, with the results from QCD sum rules in ref. [17].

6.1 Extrapolation of lattice results to the B meson.

The extrapolation to B meson needs two steps. i) For fixed \vec{q} , it is possible from lattice, varying the heavy quark mass, to fit the parameters of eq. (4). As an example we quote in table 2 the results of ref. [26], also illustrated in figure 2. Some coefficients δ in table 2 are large, indicating possible large corrections to scaling. however, in view of the large errors, due to the limited statistics in ref. [26], it is impossible to draw any firm conclusion except that *such an analysis is feasible with better statistics*. Soon, the statistics will improve, thanks to dedicated computers. This step will lead to predictions for the B meson form factors in the vicinity of q_{max}^2 . ii) The extrapolation of the B meson form factors down to $q^2 = 0$ raises yet unsolved theoretical problems as we have argued in

Figure 2: *Form factors extrapolated to the chiral limit for the light quark, as a function of the inverse pseudoscalar mass ($1/M_P$) for the momentum assignment $\vec{q} = (2\pi/L, 0, 0)$. The crosses are the lattice points, the diamonds are the extrapolation to the D and B meson. Notice that the D is very close to a lattice point. The points corresponding to the lightest heavy quark mass (furthest to the right) has not been used in the fits.*

section 1.1. Indeed, even if we take for granted from lattice results that the nearest pole dominance is not a bad approximation in $D \rightarrow K, K^*$, it does not allow to assume its validity much further away from the pole, as is the case in $B \rightarrow \pi$ decay away from q_{max}^2 . Remember that the range in q^2 is very large and one cannot be satisfied with a rough extrapolation. On the other hand, a *direct* access to B meson form factors near $q^2 = 0$ needs tiny lattice spacing ($m_b \ll a^{-1}$), i.e. a formidable increase in computer capacities.

7 NUMERICAL COMPARISON OF THE RESULTS FROM DIFFERENT APPROACHES

7.1 D meson decay.

The results are summarised in tables 1 and 3. All the predictions concerning $f^+(0)$ are compatible with experiment. For conserved currents ($m_c = m_s$)

Ref.	$\Gamma(D \rightarrow K)/10^{10}s^{-1}$	$\Gamma(D \rightarrow K^*)/10^{10}s^{-1}$	$\Gamma(D \rightarrow \pi)/10^{10}s^{-1}$
Lat. [26]	6.8 ± 3.4	6.0 ± 2.2	0.56 ± 0.36
Lat. [23]	5.8 ± 0.15	5.0 ± 0.9	0.5 ± 0.2
exp. [30, 18]	7.0 ± 0.8	4.0 ± 0.7	$0.9^{+0.5}_{-0.3}$
Ref.	$\Gamma(D \rightarrow \rho)/10^{10}s^{-1}$	$\Gamma(D \rightarrow K^*)/\Gamma(D \rightarrow K)$	Γ_L/Γ_T
Lat. [26]	0.50 ± 0.23	0.92 ± 0.55	1.27 ± 0.29
Lat. [23]	0.4 ± 0.09	0.86 ± 0.22	1.51 ± 0.27
QM [12]	—	1.14	0.89
QM [11]	—	1.45	1.11
SR [17]	—	0.5 ± 0.15	0.86 ± 0.06
Exp. [31]	—	0.63 ± 0.09	—
exp. [30]	—	0.57 ± 0.08	1.15 ± 0.17

Table 3: *Semileptonic partial widths for $D \rightarrow K$, K^* , π and ρ , using $V_{cs} = 0.975$ and $V_{cd} = 0.222$. We also report the ratio of the longitudinal to transverse polarisation partial widths for $D \rightarrow K^*$. The conventions in the first column are the same as in table 1.*

$f^+(0) = 1$. Quark model and VMD help to understand why for $D \rightarrow K$ the result is smaller than 1: For $q^2 = q_{max}^2$ the mass difference replaces the norm of the wave function, equal to 1, by an overlap of two different wave functions, smaller than 1; the nearest pole dominance implies a further decrease from q_{max}^2 down to $q^2 = 0$, further away from the pole. In quark model A_1 is very similar to f^+ (the quark spin operator replaces the identity operator). Lattice and QCD sum rules also predict A_1 not too different from f^+ , although the A_1/f^+ ratio is slightly smaller in the latter models than in quark models. V/A_1 is rather large, 1 to 2 in all cases. A_2/A_1 is of order 1 in quark models and QCD sum rules, smaller in lattice. For A_2 the situation is unclear. E691 and ref.[23] suggest a smaller value of A_2 than E653, ref.[24] and [26], even though the errors on this quantity are so large that all the results are compatible. Altogether, in view of the errors still present in theory and experiment, it is difficult to see more than an overall compatibility. No clearcut discrepancy is visible. It is worth here to stress that a better accuracy is needed on both theoretical and experimental sides. For the latter, *a TCF is really necessary*. The small discrepancies add up and are enhanced in table 3 where the partial widths are reported. The ratio $\Gamma(D \rightarrow K^*)/\Gamma(D \rightarrow K)$ is predicted too large by quark models, as we have already stated in section 3.1. QCD sum rules predict a small number, while lattice are in between, with a large error.

Ref.	$f^+(0)$	$V(0)$	$A_1(0)$	$A_2(0)$
Lat. [26]	0.30 ± 0.17	0.39 ± 0.16	0.23 ± 0.06	0.43 ± 0.28
EL [20]	0.53	0.62	0.21	0.20
QM [12]	0.33	0.33	0.28	0.28
QM [11]	0.09	0.27	0.05	0.02
SR [18]	0.26 ± 0.02	0.6 ± 0.2	0.5 ± 0.1	0.4 ± 0.2
SR [27]	0.23 ± 0.02	0.47 ± 0.14	0.35 ± 0.16	0.42 ± 0.12
Ref.	—	$V(0)/A_1(0)$	$A_2(0)/A_1(0)$	—
Lat. [26]	—	1.5 ± 0.6	1.9 ± 1.1	—
[12]	—	1.0	1.0	—

Table 4: *Semileptonic form factors for $B \rightarrow \pi$ and ρ . The notations are the same as in table 1. “EL” refers to effective Lagrangians.*

7.2 B meson decay.

Some results are reported in table 4. One first remark is that for most of the form factors the predictions of ref.[11] are much lower than all the others, cf. table 4. This results in a much larger estimate of $|V_{ub}|$, for a given experimental branching ratio. The reason for that has been discussed in section 3.2: in ref.[11] the form factor is computed at q_{max}^2 and then a “tempered” exponential dependence on q^2 is assumed. This q^2 yields a dramatic suppression at small q^2 for B meson decays, where the range in q^2 is very large. From eq. (4) the ratio A_2/A_1 scales like M_P at fixed \vec{q} . The q^2 dependence being assumed to be rather smooth, we expect $A_2(0)/A_1(0)$ to increase with M_P , although $|\vec{q}| \sim M_P/2$. Table 4, compared with table 1 shows that indeed lattice QCD predicts $A_2(0)/A_1(0)$ larger for B decay than for D decay. Neither QCD sum rules, ref. [18], nor the quark model of ref. [12] show this behaviour. Altogether table 4 shows a wide spreading of the predictions for the B semileptonic form factors. This becomes even worst when one considers the partial widths. A summary is given in table IV in ref. [18]. The outcome is that in units of $|V_{ub}|^2 10^{13} s^{-1}$ the predictions for $\Gamma(B \rightarrow \pi)$ range from 0.3 to 1.45 for QCD sum rules, from 0.21 to 0.74 for quark models, to which we add the effective Lagrangian of ref.[20] prediction¹⁰ of 1.9 and the lattice prediction of ref.[26]: 1.2 ± 0.8 . For $\Gamma(B \rightarrow \rho)$, in the same units, 0.77 to 3.3 from QCD sum rules, 1.63 to 2.6 from quark models, 2.12 from effective Lagrangian, and 1.3 ± 1.2 from lattice. The conclusion is simple: *today, nothing reliable is known about the magnitude of B meson exclusive semileptonic decays.*

¹⁰ Concerning effective Lagrangian, we quote directly the numbers presented by N. Di Bartolomeo in our working group. They differ from what is reported in ref. [18].

8 USE OF THE FINAL HADRONS

Although no work has yet been done to our knowledge on these issues, it should be kept in mind that charm semileptonic decay $D \rightarrow K^{**}l\nu$ is a clean factory for K^{**} , where by K^{**} we mean any excited strange meson, heavier than $K^*(892)$. There would be a rich opportunity to increase our understanding of strange meson spectroscopy if a TCF was to be built. Next, the semileptonic decay $D \rightarrow K\pi l\nu$, with the $K\pi$ invariant mass below 892 MeV, is a unique, model independent, access to $K\pi$ phase shift, analogous to what has been done about $\pi\pi$ phase shift from the K semileptonic decay into two pions. This prospect has been repeatedly stressed by Jan Stern.

9 CONCLUSION

An accurate knowledge of CKM angles is necessary to understand the Standard Model mass matrix, the generation puzzle. Such a knowledge needs a b -factory. But without a theoretical knowledge of B matrix elements, to the wanted accuracy, a b -factory would be useless. There is no better testing band of the models, methods and ideas used to estimate these matrix elements, than charm decays, whose relevant KM angles are already known to a sufficient accuracy. If nature had provided us with a heavier charm, closer to the b , life would be easier, we would not need such a distant extrapolation. But we have no choice, we have to manage with the existing charm and to perform a long distance extrapolation. We have to learn the corrections to scaling laws when the heavy mass varies. We have to learn how the form factors depend on q^2 . This is the challenge to theorists. Under this proviso, *a TCF and a b -factory appear not as competitors but as complementary facilities*, necessary to reach both a better understanding of non perturbative QCD and a better accuracy on CKM angles.

References

- [1] N. Isgur and M.B. Wise, Phys.Rev. D42 (1990) 2388.
- [2] I.I. Bigi, M. Shifman, N.G. Uraltsev and A. Vainshtein, UND-HEP-93-BIG01.
- [3] T. Altomari and L. Wolfenstein, Phys.Rev.Lett. 58 (1987) 1583.
- [4] E. Golowich, F. Iddir, A. Le Yaouanc, L. Oliver, O. Pène and J.C. Raynal, Phys.Lett. B213 (1988) 521.
- [5] N.Isgur and M.Wise, Phys.Lett.B232 (1990) 113; Phys. Lett. B237 (1990) 527; Phys. ReV. Lett. 66 (1991) 1130; H.Georgi and F.Uchiyama, Phys.

- Lett. B238 (1990) 395; H.Georgi and M.Wise, Phys. Lett. B243 (1990) 279; J.D. Bjorken, 25th Int. Conf. on High Energy Physics, Singapore, Aug 2-8, 1990, p 329; H.Georgi, Nucl.Phys. B361 (1991) 339.
- [6] J.D. Bjorken SLAC Summer Inst. 1990 p 167.
 - [7] A. Le Yaouanc et al., Gif lectures 1991 (electroweak properties of heavy quarks), tome 1, p89.
 - [8] M.E. Luke, Phys.Lett. B252 (1990) 447.
 - [9] A. Le Yaouanc et al. Nucl.Phys. B37 (1972) 552.
 - [10] F.E. Close and A. Wambach, RAL-93-022, may-june 93.
 - [11] N. Isgur, D. Scora, B. Grinstein and M.B. Wise, Phys.Rev. D39 (1989) 799; N. Isgur and D. Scora Phys.Rev. D40 (1989) 1491.
 - [12] M. Bauer, B. Stech and M. Wirbel, Z. Phys. C29 (1985) 637; C34 (1987) 103.
 - [13] A.L. Licht and A. Pagnamamenta, Phys.Rev. D2 (1970) 1150 and 1156.
 - [14] Körner and Schuler Z. Phys. C38 (1988) 511; C41 (1989) 690.
 - [15] G.Martinelli, These proceedings.
 - [16] P. Colangelo, G. Nardulli and N. Paver BARI-TH/93-132, bulletin board hep-ph/9303220.
 - [17] P. Ball, V.M. Braun, H.G. Dosch Phys.Rev. D44 (1991) 3567.
 - [18] P. Ball, TUM-T31-39-93, Bulletin Bd.: hep-ph@xxx.lanl.gov - 9305267.
 - [19] T.M. Aliev, A.A. Ovchinnikov and V.A. Slobodenyuk, Trieste Preprint IC/89/382 (1989).
 - [20] R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, R. Gatto, F. Feruglio, G. Nardulli Phys.Lett. B299 (1993) 139.
 - [21] M.Crisafulli et al., Phys.Lett. 223B (1989) 90.
 - [22] V.Lubicz, G.Martinelli and C.T.Sachrajda, Nucl.Phys. B356 (1991) 310.
 - [23] V.Lubicz, G.Martinelli, M.McCarthy and C.T.Sachrajda, Phys.Lett. 274B (1992) 415.
 - [24] C.Bernard, A.El-Khadra and A.Soni, Phys.Rev.D43 (1992) 2140..

- [25] C. Bernard, A. El-Khadra and A. Soni, Phys.Rev. D45 (1992) 869.
- [26] As. Abada et al. LPTENS 93/14.
- [27] S. Narison, Phys.Lett. B283 (1992) 384.
- [28] J.C. Anjos et al. Phys.Rev.Lett. 65 (1990) 2630.
- [29] K. Kodama et al. Phys.Lett. B274 (1992) 246.
- [30] See for example S.Stone, in Heavy Flavour Physics, eds. A.J. Buras and H. Lindner, world scientific, Singapore (1992).
- [31] CLEO II, as reported by P. Burchat, 5th International Symposium on Heavy Flavour Physics, Montreal, Canada, 6-10 July 1993, to appear in the proceedings.



